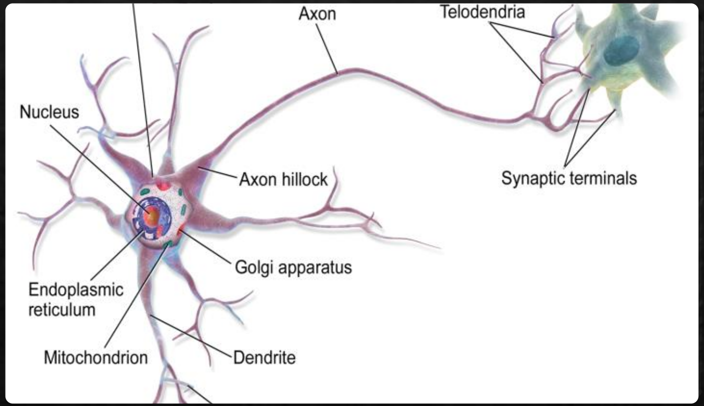
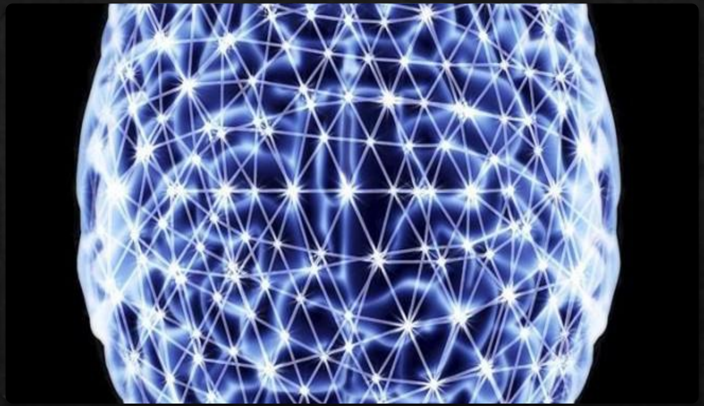
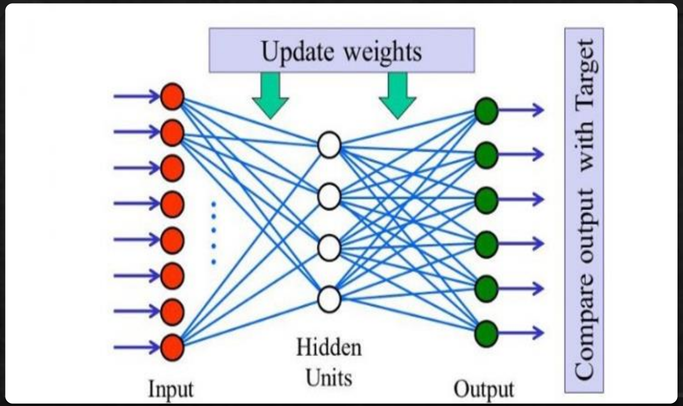
Neuroscience

Ata Tahuri

Neuroscience







Biology

Physics

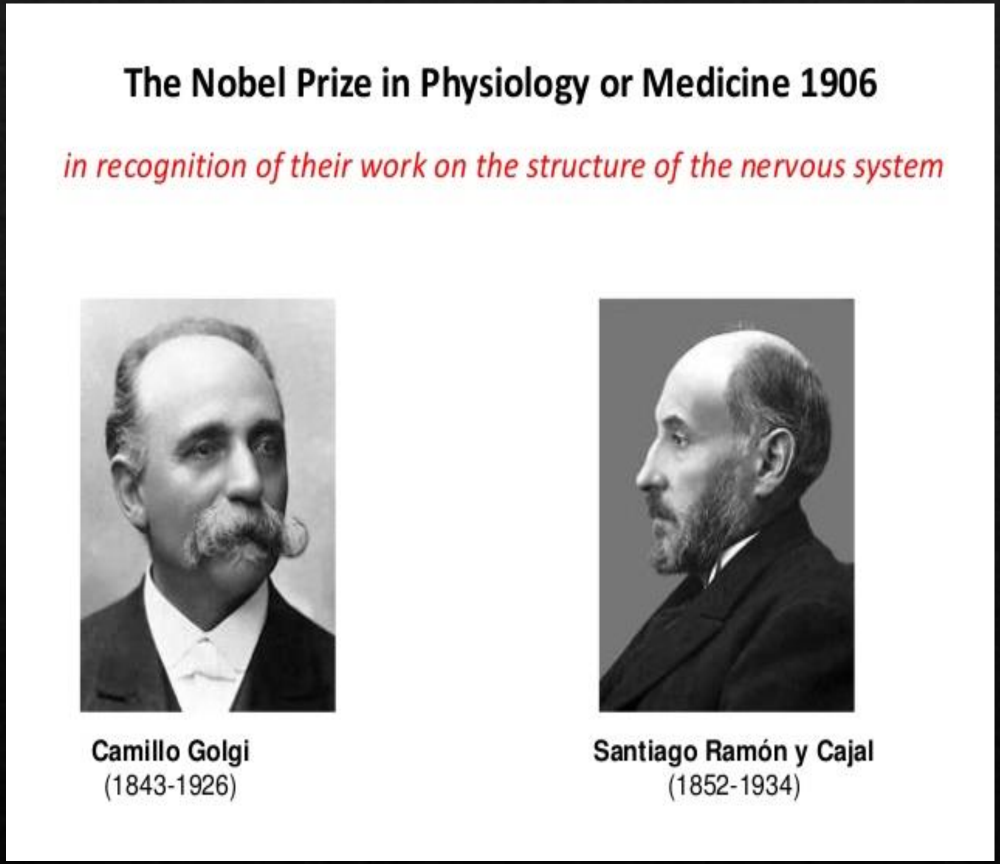
Philosophy

Computer science

Mathematics

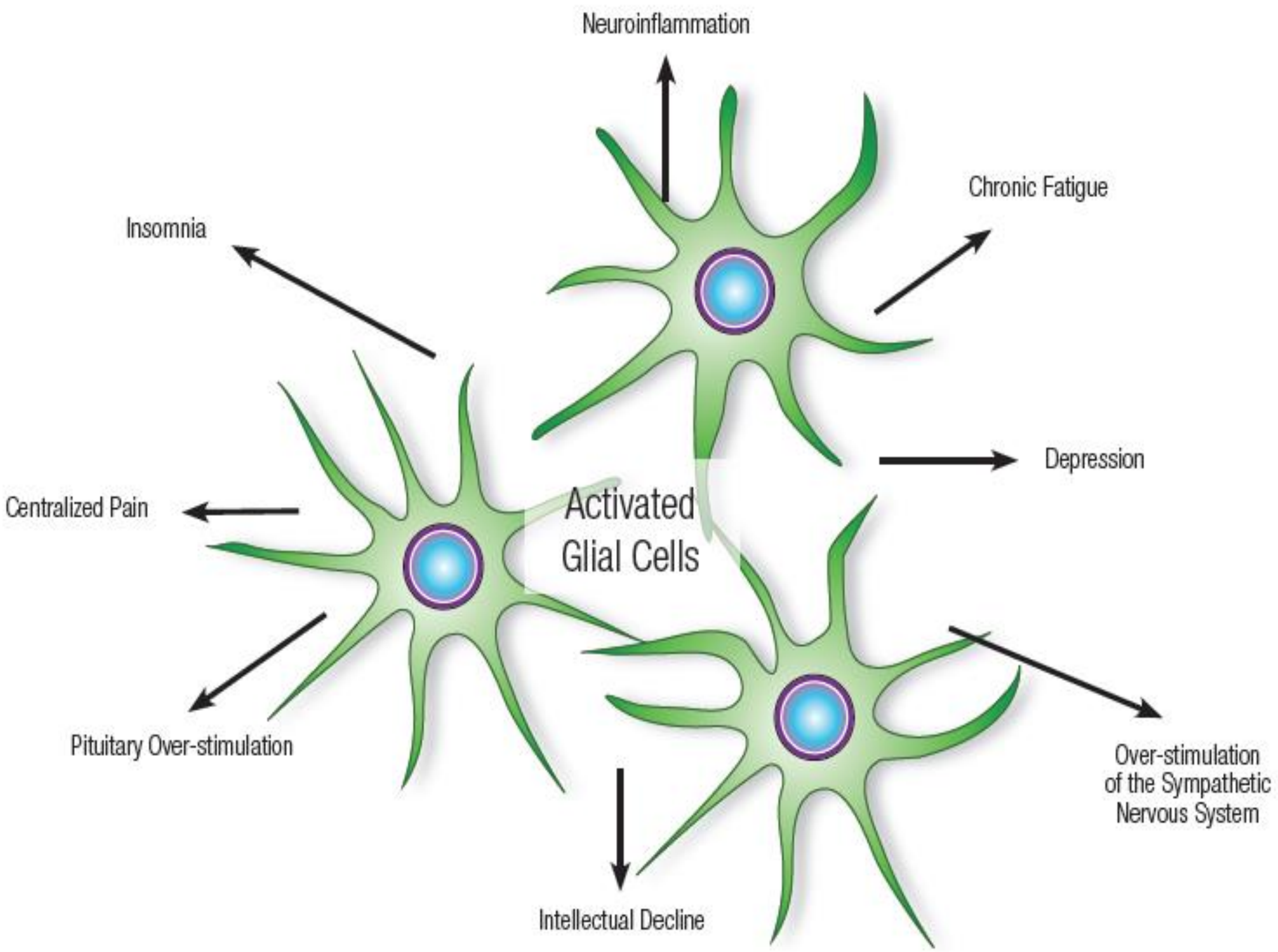
Electrical Engineer

- Biology : How do we think ?/Which kind of chemical and biological actions are happening in our brain?



Cajal

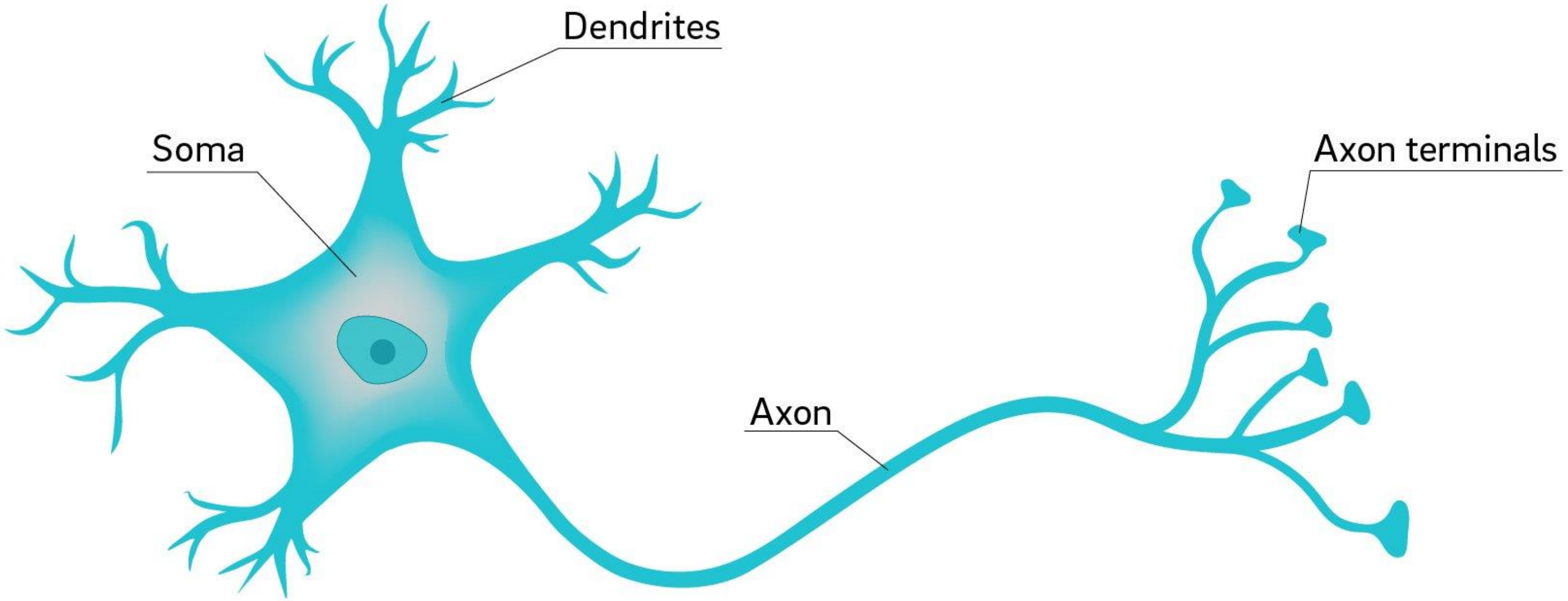
Golgi

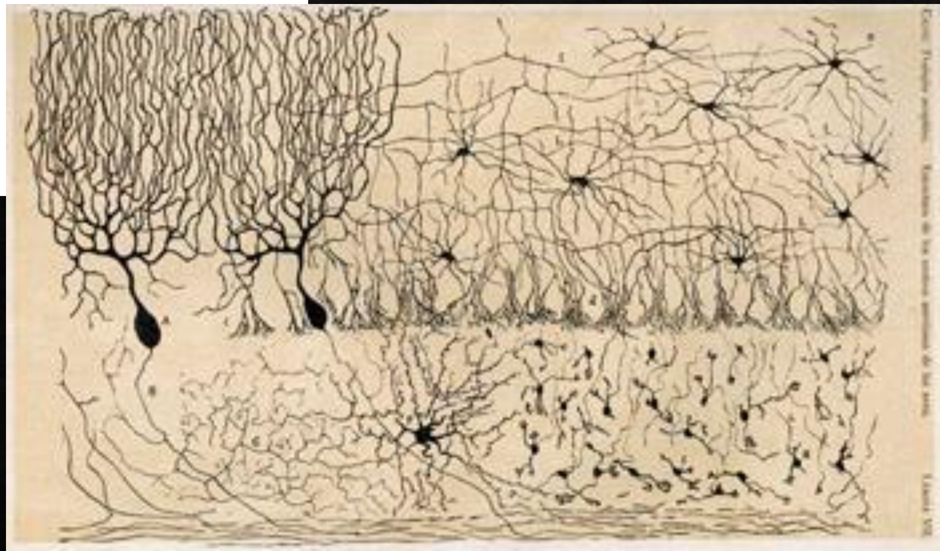


Glial cells

- Immune
- Nutrition
- Micro glial

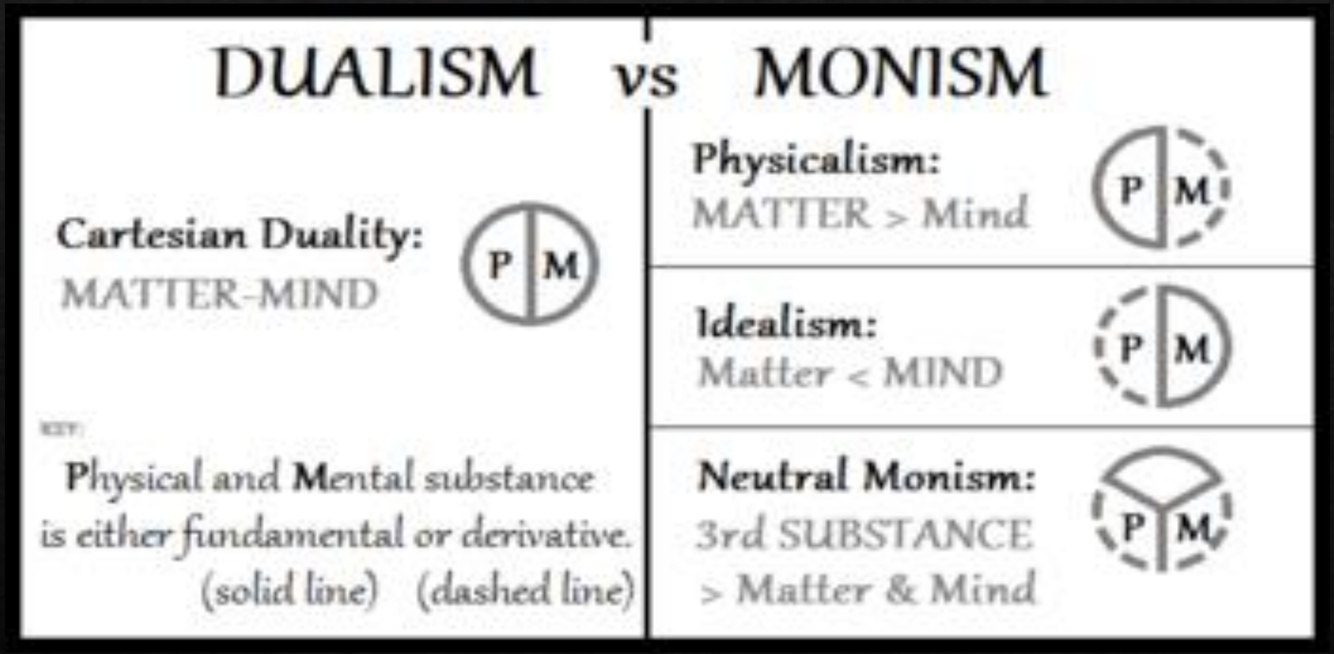
Neuron



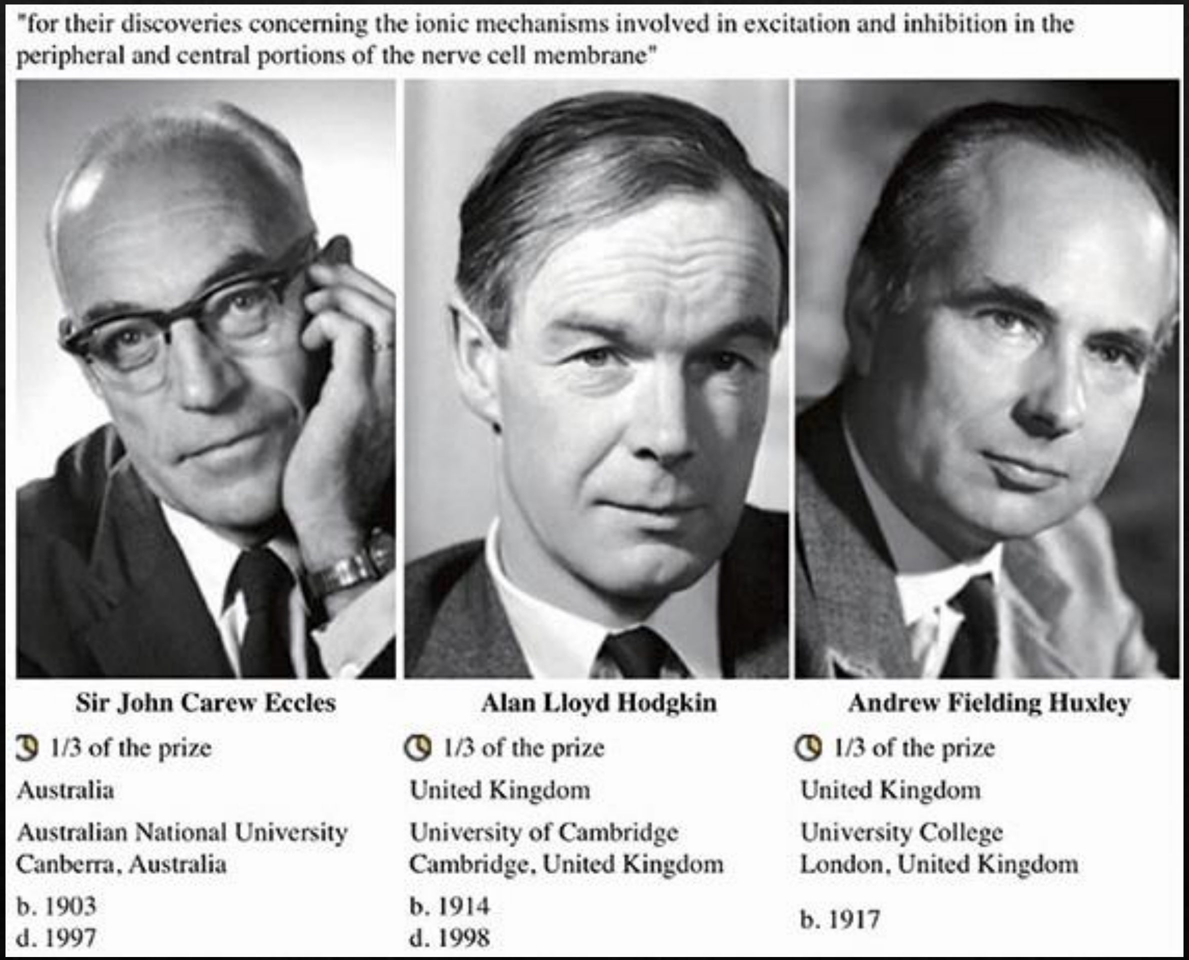


- Philosophy: Monism vs Dualism / Perception(brain vs Spirit)

What is the relationship between body and mind?



- Math: How dynamics of neurons are controlling us?/ Are we just complicated circuits?



- Physics: Ising model/Can physical laws control us as they govern universe?

Ising

- Computer science: Are we advanced computer or not?/Is NP = P ?

Alan Turing

Mathematical model

- Hodgkin-Huxley model(ODE)
- Reducing continuous model to discrete model by D.Terman
- Discrete model of neuronal dynamics
- Searching for properties of discrete model
- Boolean Functions and gene regulatory
- Olfaction

Discrete Model

Definition1. A *directed graph* or *digraph* is a pair $D = <V_D, A_D>$, where $A_D \subseteq V_D^{\ 2}$. The set V_D is called the set of *vertices* or *nodes* of D, and the set A_D is called the set of *arcs* of D.

Definition2. A *neuronal network* is a triple $N = <D, p, th>$, where $D =[n]$, AD is a loop-free digraph and $p = (p_1, . . . , p_n)$ and $th = (th_1, . . . , th_n)$ are vectors of positive integers. The state s of the system at time t is a vector $s(t) = (s_1(t), . . . , s_n(t))$, where $s_i (t) \in \{0, 1, . . . , p_i \}$ for all $i \in [n]$. The state space of N will be denoted by St_N . The dynamics of N is defined as follows:

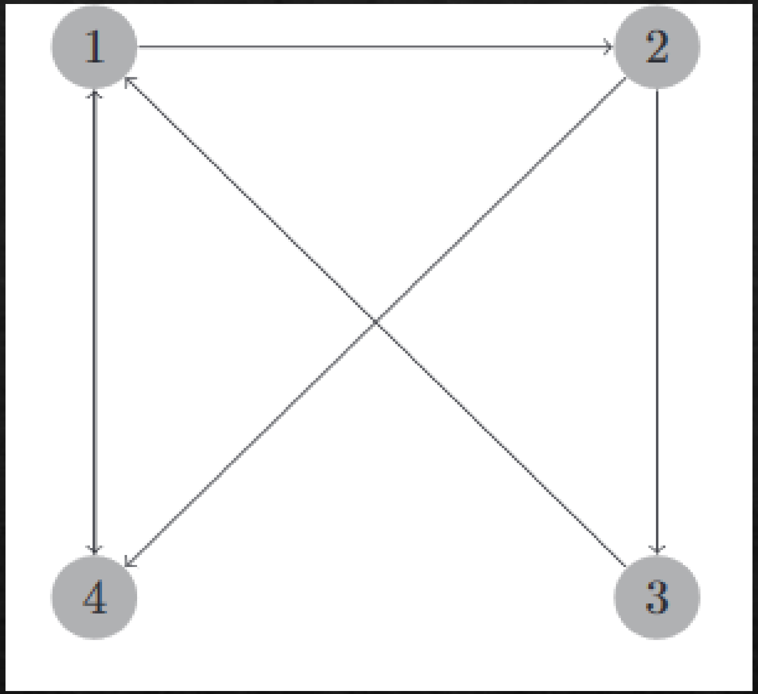
- If $s_i(t) < p_i$, then $s_i (t + 1) = s_i(t) + 1$.
- If $s_i(t) = p_i$ and there exist at least th_i different $j \in [n]$ with $s_j (t) = 0$ and $j, i \in AD$, then $s_i (t + 1) = 0$.
- If $s_i (t) = p_i$ and there are fewer than th_i different $j \in [n]$ with $s_j (t) = 0$ and $j, i \in AD$, then $s_i(t + 1) = p_i$.

This page intentionally left blank

Example.

$p = (1, 1, 1, 2)$, $th = (2, 1, 1, 1)$

- $s(0) = (0, 1, 1, 1)$
- $s(0) = (1, 0, 0, 0)$
- $s(0) = (0, 1, 1, 2)$.



Definition3. The ***trajectory*** of the state $S(t)$ is the sequence $\{S(t+1), S(t+2), ...\}$.

Definition4. Since state space is finite, we must eventually have $S(t_0) = S(t_1)$ for some $0 < t_0 < t_1$. Therefore from time t_0 the sequence would be cyclic. Let t_0, t_1 be the smallest times with this property. Then the set $\{S(t_0), ..., S(t_1 - 1)\}$ is called the ***attractor*** of $S(0)$. its length is $t_1 - t_0$.

Note that the attractor of length one is called *steady state*. Can we determine all steady states in a network?

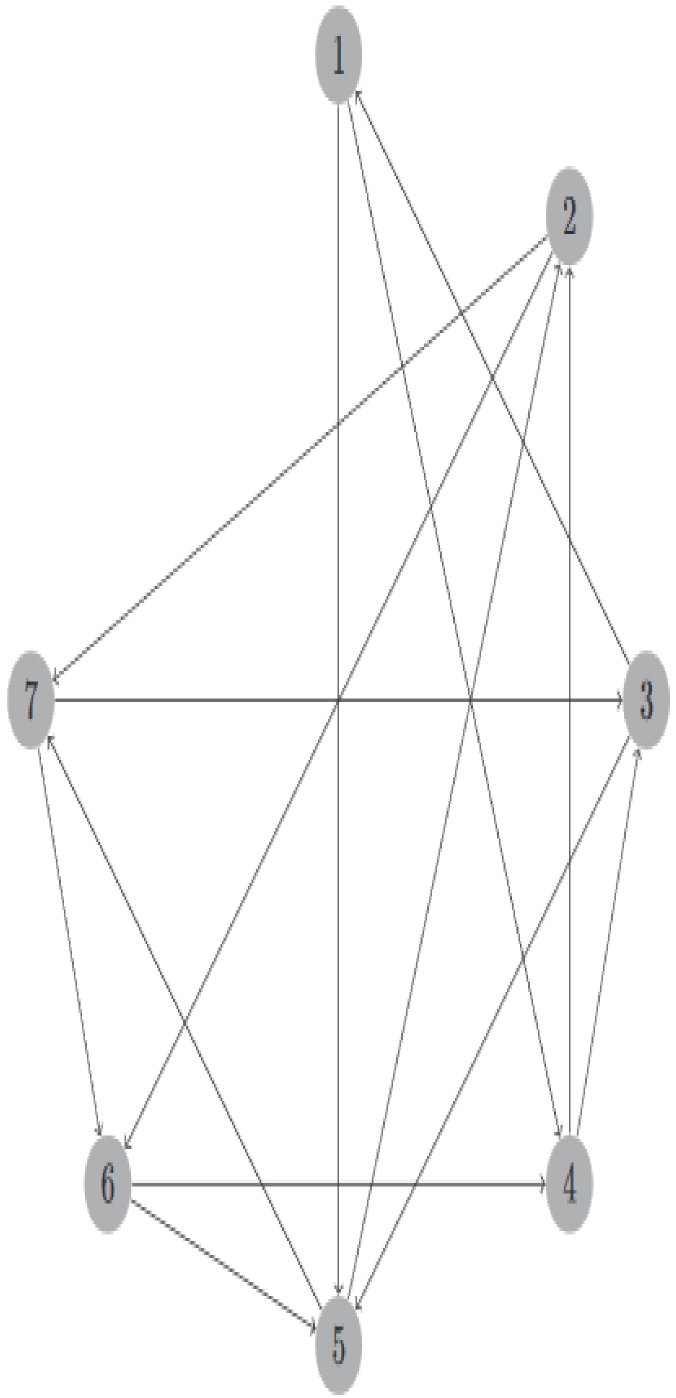
Definition5. The set $\{S(0), ..., S(t_0 - 1)\}$ is called the ***transient*** of $S(0)$. The length of transient is t_0. Note that a transient of length zero occurs whenever the initial state is in an attractor.

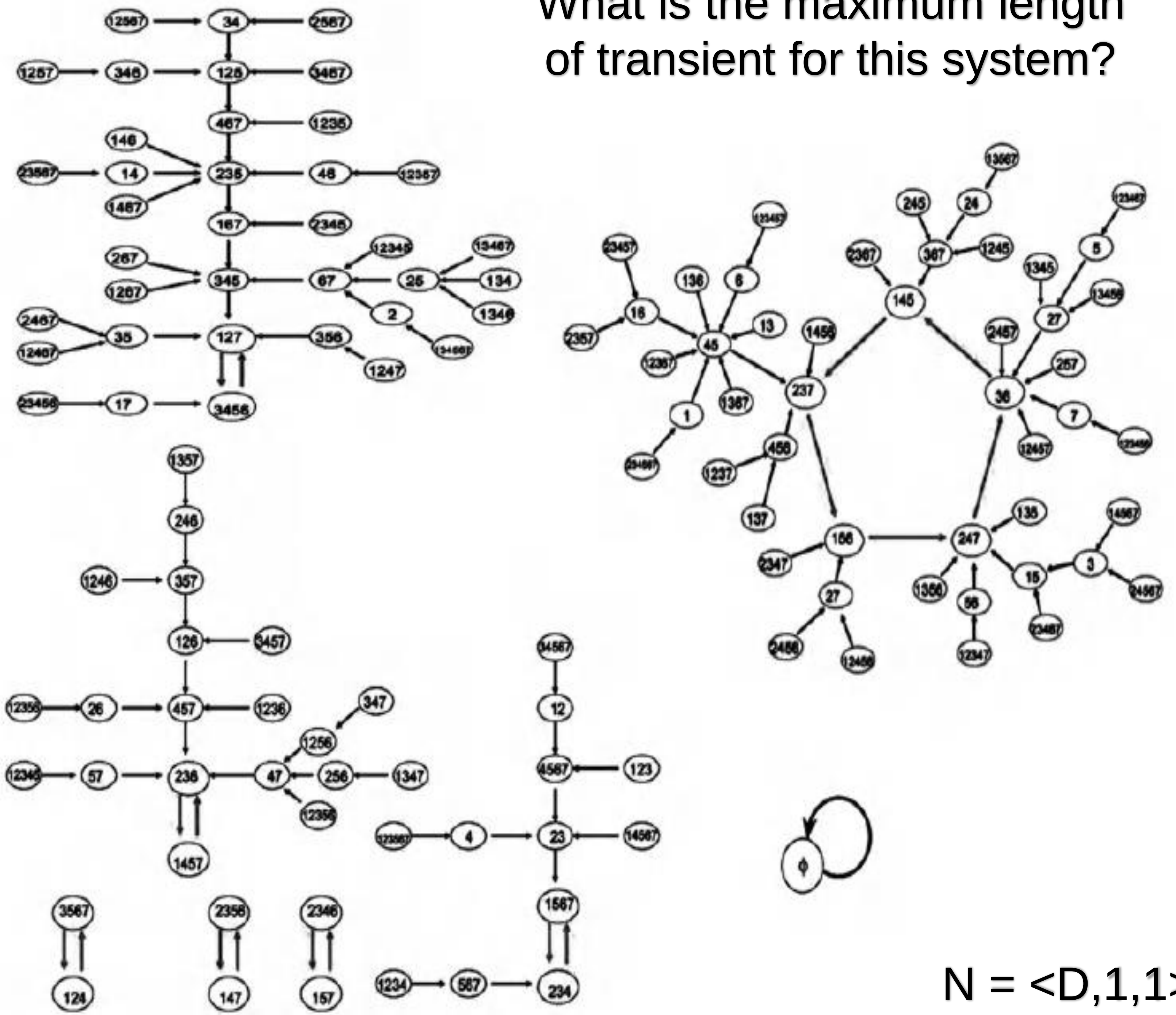
Definition6. For any neuronal network $N = \langle D, p, th \rangle$ we can define another digraph D_N whose vertices are the states and whose arcs indicate the *successor state* for each state, that is, s, s^* is an arc in this digraph if and only if $s(t) = s$ implies $s(t+1) = s^*(t)$.

This digraph will be called the *state transition digraph* of the network.

This page intentionally left blank

What is the maximum length of transient for this system?





N = <D,1,1>

Definition7. A digraph D is *acyclic* if D does not contain any directed cycle.

Lemma. *Let $N = <D, p, th>$ be a neuronal network whose connectivity D is acyclic. Then $\{p\}$ is the only attractor in N.* *Proof: ?*

Theorem . *Let $N = <D, p, th>$ be a network whose connectivity D is acyclic. Then*

a. *the maximum length of each attractor in N is 1,*

b. *the maximum number of different attractors in N is 1,*

c. *the size of the basin of attraction of the steady state attractor is* $|St_N| = \prod_i (p_i + 1)$,

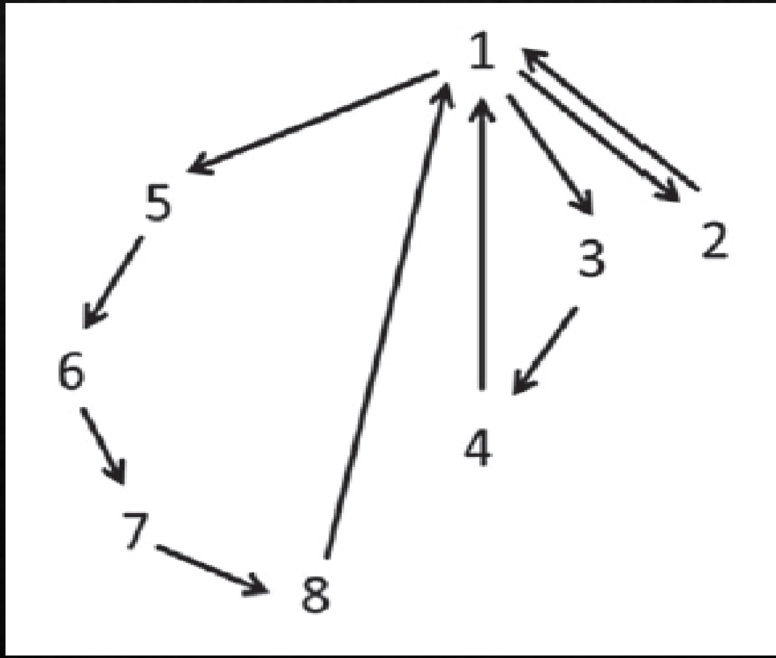
d. *the transients have length at most* $n + p_* - 1$.

Proof: ?

This page intentionally left blank

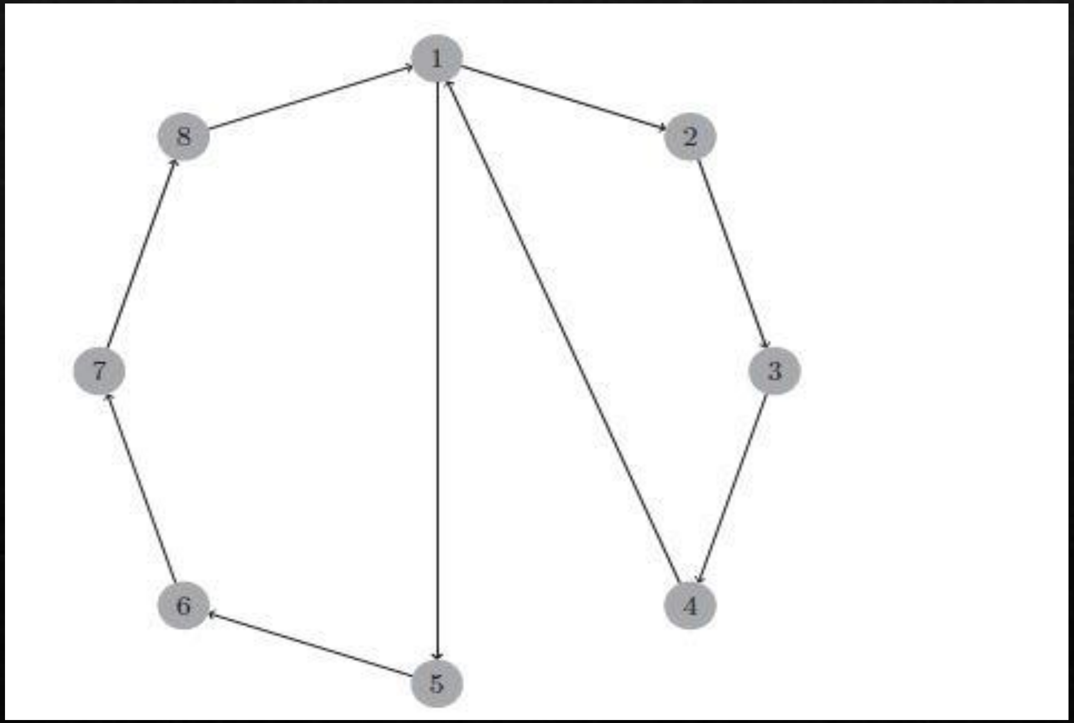
Landau's Function

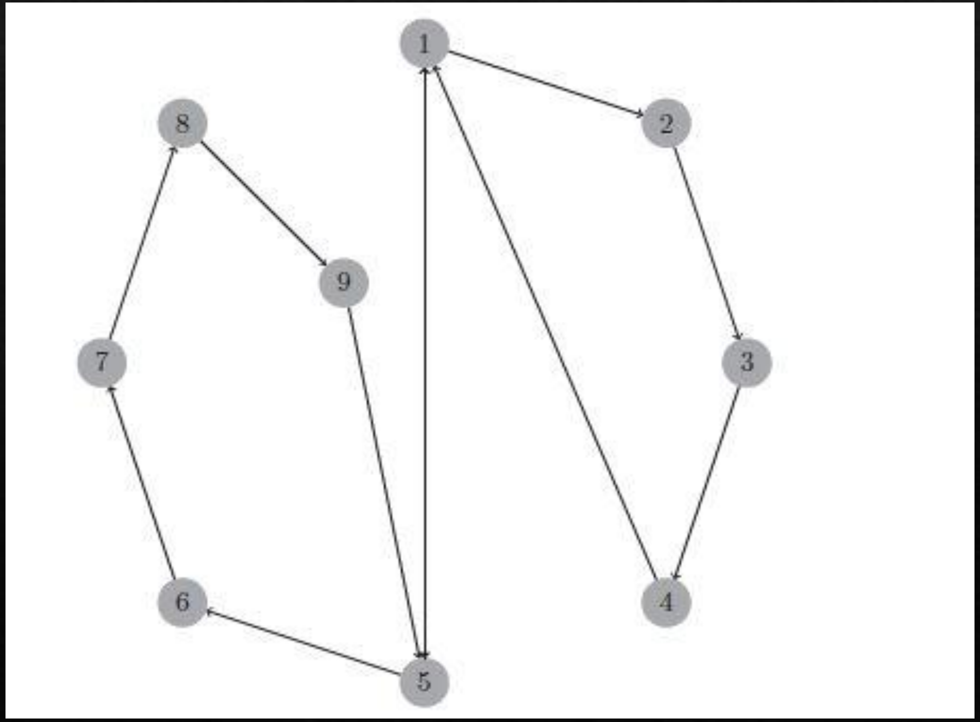
- $g(n) = \max \operatorname{lcm}_{n_1+\ldots+n_{m<n}}\{n_1,\ldots,n_m\}$.
- g(n) grows approximately like $e^{\sqrt{n \ln n}+O(1)}$.



Conjecture

Let N = $\langle D, 1,1 \rangle$ be a network such that D is a strongly connected digraph on n. If n < 26, then N does not have attractors of length > n.





$A_D = D_C(26,10) \cup \{<25,11>\}$

Act_0	2	5	8	10			13	15	18	21	23	25	
Act_1	1	3	6	9		11		14	16	19	22	24	26
Act_2		2	4	7	10		12		15	17	20	23	25
⋮							⋮						
Act_{28}	3	6	8	10		11	13	16	19	21	23		26
Act_{29}	1	4	7	9			12	14	17	20	22	24	
Act_{30}		2	5	8	10			13	15	18	21	23	25

Thank you