فلسفه ریاضیات

۲۲-۱۲۶

نیمسال دوم ۱۴۰۳-۱۴۰۲

دانشکده علوم ریاضی، دانشگاه صنعتی شریف

A. Introductory Texts

- **Bostock, D.** *Philosophy of Mathematics: An Introduction* (2009)
- **Brown, J. R.** *Philosophy of Mathematics*, 2nded (2008)
- **Çevik, A.** *Philosophy of Mathematics: Classic and Contemporary* Studies (2022)
- **Colyvan, M.** *An Introduction to the Philosophy of Mathematics* (2011)
- **Friend, M.** *Introducing Philosophy of Mathematics* (2007)
- **George, A. and D. Velleman** *Philosophies of Mathematics* (2002)
- **Hamkins, J. D.** *Lectures on the Philosophy of Mathematics* (2020)
- **Körner, S.** *The Philosophy of Mathematics: An Introductory Essay* (1960)
- **Linnebo, Ø.** *Philosophy of Mathematics* (2017)
- **Ravn, O. and O. Skovsmose** *Connecting Humans to Equations* (2019)
- **Shapiro, S.** *Thinking about mathematics: The philosophy of mathematics* (2000)

- *Recommended as course texts*
- *Old but good*
- *Good, philosophy oriented*

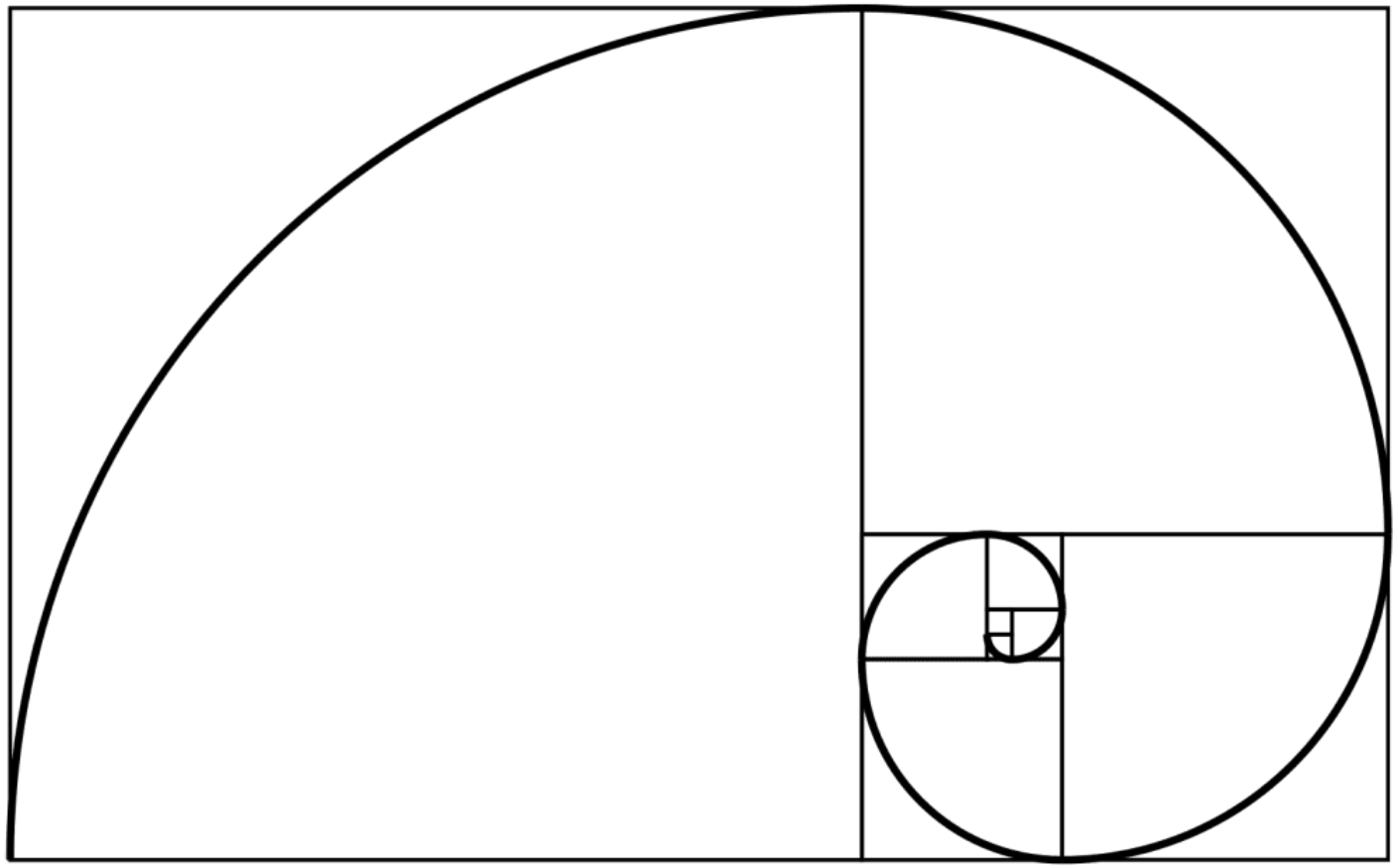
B. General Collections of Essays

- Benacerraf, P. and H. Putnam *Philosophy of Mathematics: Selected Readings*, 2nded (1983)
- Dybjer, P. et al *Epistemology versus Ontology* (2012)
- Ewald, W. *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Vols. 1&2 (1996)
- Hart, W. *The Philosophy of Mathematics* (1996)
- Irvine, A.D. *Philosophy of Mathematics* (2009)
- Lindström, S., et al *Logicism, Intuitionism and Formalism: What has become of them?* (2009)
- Shapiro, S. *The Oxford Handbook of the Philosophy of Mathematics and Logic* (2005)

Chronology: `Greek' Mathematics

Thales	-590	Euclid	-300
Pythagoras	-530	Archimedes	-250
Theaetetus	-390	Apollonius	-220
Plato	-380	Nicomachus	+90
Eudoxus	-370	Diophantus	+250
Aristotle	-350	Pappus	+300

مستطیل طلایی



مراجع اصلی افلاطون

- ***Collected Dialogues, including the Letters***

  Hamilton & H. Cairns (Eds.), Princeton U. Press 1963

- Wedberg, A. ***Plato's Philosophy of Mathematics***

  Almquist and Wiksell 1955

گفتگوهای اصلی مربوط به ریاضیات و معرفت شناسی

- **Meno, Republic**
- **Phaedo, Theaetetus, Parmenides, Philebus, Timaeus**
- **Gorgias, Phaedrus, Laws**

مراجع اصلی ارسطو

- ***The Basic Works of Aristotle***
  McKeon, R. (ed.), Modern Library 2001
- McKeon, R. ***Introduction to Aristotle***, Modern Library 1947
- Bostock, D. `Aristotle's Philosophy of Mathematics' in
  Oxford Handbook of Aristotle (2012)
- Mendell, H. `Aristotle and Mathematics', in
  Stanford Encyclopedia of Philosophy
  https://plato.stanford.edu/entries/aristotle-mathematics/

ارجاعات به آثار ارسطو

- ***Metaphysica*** (مابعدالطبیعه) **:** I.9, II.2, III.5-6, V.2, IX.9, X.1-3, XI.10-12, XIII.1-10, XIV.1-6
- ***Physica*** (طبیعیات) **:** II.2-3, III.4-8, V.3, VI.1-10
- ***Categoriae*** (مقولات) : V.6
- ***Analytica Priora*** (منطق) : I.1-13, 27-30
- ***Analytica Posteriora*** (آنالوطیقای‌دوم) **:** I.1-2, I.7, I.26
- ***De Anima*** (روح) **:** III.3, III.7
- ***De Caelo*** (سماوات) **:** I.2, I.5-7, II.13-14

ماهیت اشیاء ریاضی

- The faculty of thinking then thinks the forms in images…the mind when it is thinking the objects of Mathematics thinks as separate, elements that do not exist as separate. In every case the mind which is actively thinking is the objects which it thinks. ***De Anima*****:** III.7

- Now the mathematician, though he too treats these things, nevertheless does not treat them as limits of physical body; nor does he consider the attributes indicated as the attributes of such bodies. That is, he separates them; for in thought they are separable from motion, and it makes no difference, nor does any falsity result, if they are separated. ***Physica***: II.2

Analytica Posteriora, I.2 مبنای دانش علمی

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in an accidental way… when we know the cause on which the fact depends…
What I now assert is that at all events we do know by demonstration. I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is *eo ipso* such knowledge. Assuming then my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause… The premises must be true: for that which is non-existent cannot be known – we cannot know, e.g., that the diagonal of a square is commensurable with side.

Analytica Posteriora, I.2

مبنای دانش علمی

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in an accidental way… when we know the cause on which the fact depends…What I now assert is that at all events we do know by demonstration. I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is *eo ipso* such knowledge. Assuming then my thesis as to the nature of scientific knowing is correct, the premises of demonstrated knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to cause…

Discrete vs Continuous: *Categoriae*, V.6

Quantity is either discrete or continuous…Instances of discrete are numbers and speech; of continuous, lines, surfaces, solids, and, besides these, time and place…
In the case of the parts of a number, there is no common boundary at which they join. For example: two fives make ten, but the two fives have no common boundary, but are separate; the parts three and seven also do not join at any boundary… Number, therefore, is a discrete quantity…A line, on the other hand, is a continuous quantity, for it is possible to find a common boundary at which its parts join. In the case of the line, this common boundary is the point…

Discrete vs Continuous: *Analytica Posteriora*, I.7

It follows that we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. For there are three elements in demonstration: (1) what is proved, the conclusion – an attribute inhering essentially in a genus; (2) the axioms, i.e., the axioms which are premises of demonstration; (3) the subject genus whose attributes, i.e., essential properties, are revealed by the demonstration. The axioms which are the premises of demonstration may be identical in two or more sciences: but in the case of two different genera such as arithmetic and geometry you cannot apply arithmetical demonstration to the properties of magnitudes unless the magnitudes in question are numbers…(cont.)

Discrete vs Continuous: *Analytica Posteriora*, I.7

Arithmetical demonstration and the other sciences likewise possess, each of them, their own genera; so that if the demonstration is to pass from one sphere to another, the genus must be either absolutely or to some extent the same. If this is not so, transference is clearly impossible, because the extreme and the middle terms must be drawn from the same genus: otherwise as predicated, they will not be essential and will thus be accidents. That is why it cannot be proved by geometry that …the product of two cubes is a cube.

Summary Contents of Euclid's *Elements*

Book	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	Totals
Definitions	23	2	11	7	18	4	22	–	–	16	28	–	–	131
Postulates	5	–	–	–	–	–	–	–	–	–	–	–	–	5
Common Notions	5	–	–	–	–	–	–	–	–	–	–	–	–	5
Propositions	48	14	37	16	25	33	39	27	36	115	39	18	18	465

https://en.wikipedia.org/wiki/Euclid%27s_Elements
(February 9,2024)

منابع ریاضیات دوره اسلامی

- منابع اولیه به عربی و ترجمه‌های فارسی و زبان‌های اروپایی
- مقالات متعدد به‌خصوص از Roshdi Rashed و Jan Hogendijk
- از دوکتاب زیر خیلی استفاده شده است:

Rashed, R. ***The Development of Arabic Mathematics: Between Arithmetic and Algeba***,1994

Rashed,R. & B. Vahabzadeh

Omar Khayyam, the Mathematician, 2000

مطالب این کتاب شامل محاسباتی است درارث ووصیت ومقاسمه (= تقسیم کردن اموال مشترك) و اموردیوانی وتجارت، و نیز درمورد تمام اموری که به‌حساب ومعامله مربوط می‌شود ـ مانند: مساحت کردن زمینها و اندازه گیری نهرها و هندسه (= نقشه کشی) و دیگر مباحث و فنون ریاضی ـ قابل استفاده خواهد بود. این کتاب را با حسن‌نیتی که

- از مقدمه *جبر و مقابله* خوارزمی، ترجمه حسین خدیوجم
- انتشارات اطلاعات، ۱۳۶۳

بحث ائودوکسوس (فصل پنج اقلیدس) از مقایسه دو نسبت :

A و **B** دو مقدار از یک کمیت پیوسته همگن، و **C** و **D** دو مقدار از یک کمیت پیوسته همگن (دیگر) هستند . نسبت **(A:B)** را برابر نسبت **(C:D)** می‌نامیم در صورتی که به‌ازای هر دو عدد صحیح(مثبت) **m** و **n** (حداکثر) دفعاتی که **nB** در **mA** می گنجد برابر (حداکثر) دفعاتی باشد که **nD** در **mC** می گنجد . چنانچه تعداد دفعات مربوط به **(A:B)** از تعداد دفعات مربوط به **(C:D)** کوچک‌تر باشد ، نسبت **(A:B)** ازنسبت **(C:D)** بزرگتر است.

توجه : برابری دو نسبت ناگویا در تعدادی متناهی گام نتیجه نمی‌شود .

بحث خیام از مقایسه دو نسبت :

A و **B** دو مقدار از یک کمیت پیوسته همگن، و **C** و **D** دو مقدار از یک کمیت پیوسته همگن (دیگر) هستند . برای نسبت **(A:B)** نمایش کسر مسلسل $[r_0;r_1,r_2,r_3,...]$ وبرای نسبت نمایش **(C:D)** کسر مسلسل $[s_0;s_1,s_2,s_3,...]$ را درنظر می گیریم . اگر به‌ازای هر **n** ، r_n و s_n برابر باشند می گوییم دو نسبت برابرند . فرض کنید **n** کوچکترین مرتبه‌ای باشد که مثلا $r_n < s_n$ چنانچه **n** فرد باشد، نسبت **(A:B)** از نسبت **(C:D)** بزرگتر است، و چنانچه **n** زوج باشد، نسبت **(A:B)** از نسبت **(C:D)** کوچکتر است

توجه : برابری دو نسبت ناگویا در تعدادی متناهی گام نتیجه نمی‌شود .

منابع فلسفه ریاضیات دوره اسلامی

Ardeshir, M. `Ibn Sina's Philosophy of Mathematics,' in ***The Unity of Science in the Arabic Tradition***, 2008

Zarepour,M.S. `Arabic and Islamic Philosophy of Mathematics,' in ***Stanford Encyclopedia of Philosophy***, 2002 https://plato.stanford.edu/entries/arabic-islamic-phil-math/

Wisnovsky, R. ***Aspects of Avicenna***, 2001 (Articles by D. Gutas and D.N. Hasse)

Black, D.L. `Estimation in Avicenna: The Logical and Psychological Dimensions,' ***Dialogue*** 32, 219-58 (1993)

Greek → Arabic → Latin

Thales	-590	Pappus	+320	Samaw'al	+1150
Pythagoras	-530	Eutocius	+510	Tusi, Nasireddin	+1200
Theaetetus	-390	Khwarizmi	+810	Fibonacci	+1202
Eudoxus	-370	Mahani	+820	Kashi	+1420
Euclid	-300	Karaji	+950	Cardano	+1545
Archimedes	-250	Ibn-Haytham	+1020	Viète	+1570
Apollonius	-220	Khayyam	+1090	Stevin	+1585
Diophantus	+250	Tusi, Sharafeddin	+1135	Descartes	+1637

رومی‌ها و ریاضیات

- نقل از **Cicero** :

یونانیان هندسه را بسیار ارج می‌نهادند و از اینرو برای هیچکس به اندازه ریاضی‌دانان احترام قائل نبودند، ولی رومیان این فن را به وسیله‌ای برای اندازه‌گیری و محاسبه محدود کرده‌اند .

نقل از **Plutarch** :

فن ساختن ابزار هندسی را آرخیتاس و ائودوکسوس ابداع کردند و موفق شدند بعضی ترسیمات را که با خطکش و پرگار ممکن نبود با این ابزار انجام دهند . ولی افلاطون آنها را به اتهام تنزل دادن مقام هندسه سرزنش کرد . . . به این سبب مکانیک از هندسه جدا شد و از جرگه فلسفه به یک فن نظامی مبدل گشت .

اروپای قرن شانزده

Gerolamo Cardano	***Ars Magna* (1545)** حل جبری معادلات درجه۳ و ۴، پذیرش اعداد منفی و به‌کارگیری محدود اعداد موهومی
Simon Stevin	***De Thiende* (1585), *L'arithmétique* (1585)** رواج دادن عددنویسی (کسری) اعشاری در اروپا، یکپارچه کردن مفهوم عدد حقیقی، قضیه مقدار بینی برای چندجمله‌ایها
François Viète	***Algebra Nova* (1591), *Supplementum geometriae* (1593)** آغاز استفاده کامل از جبر نمادین، پیشتاز هندسه تحلیلی

René Descartes (1596-1650)

- **Here I beg you to observe in passing that the scruples that ancient writers observed in using arithmetical terms in geometry, thus making it impossible for them to proceed beyond a point where they could see clearly the relation between the two subjects, caused much obscurity and embarrassment, in their attempts at explanation.**
- **I would borrow the best of geometry and of algebra and correct all the faults of the one by the other.**

اولین جملات کتاب هندسه دکارت:

Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction. Just as arithmetic consists of only four or five operations, namely addition, subtraction, multiplication, division and the extraction of roots, which may be considered a kind of division, so in geometry, to find required lines it is merely necessary to add or subtract other lines; or else, taking one line which I shall call unity in order to relate it as closely as possible to numbers, and which can in general be chosen arbitrarily …

مراجع دكارت:

- ***The Geometry of Rene Descartes with a facsimile of the first edition*** (Dover 1954)
- ***Discourse on the Method, Optics, Geometry and Meteorology*** (Hackett 2001)
- ***Meditations on First Philosophy*** (Cambridge 1996)