Game Theory - Week 1

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Overview

- Course Overview
- What is a game?
- Normal Form Games
- Examples of Normal Form Games
- Constant Sum Games
- Games of Cooperation
- Nash Equilibrium

Course Objectives

- Rigorous introduction to game theory and its applications to economics, political science, computer science, and biology.
- To give students a thorough understanding of how such problems are solved, and some experience in solving them
- To give students the background required to use the methods in their own research work or applications
- For more information visit course webpage https://b2n.ir/r00714

Key Ingredients of Defining Game

- Who are the decision makers?
  - People? Governments? Companies? Somebody employed by a Company?
- What can the players do?
  - Enter a bid in an auction? Decide whether to end a strike? Decide when to sell a stock? Decide how to vote?
- What motivates players?
  - Do they care about some profit? Do they care about other players?

Standard Representations of a Game

1. Normal form(Matrix form, Strategic form): Players take their actions simultaneously and payoffs are functions of players actions.
2. Extensive Form: Include timing of moves. Players move sequentially, represented as tree. Keeps track of what each player knows when he or she makes each decision

Normal Form Games

Definition (Normal Form Game)

A finite n-person normal form game is a triple $< N, A, u >$: $N = \{1, \dots, n\}$ a finite set representing players. $A = A_1 \times \cdots \times A_n$ is action profiles of players where each A_i present actions of player i and $u = (u_1, \dots, u_n)$ is a profile of utility functions where each u_i is utility function of player i.

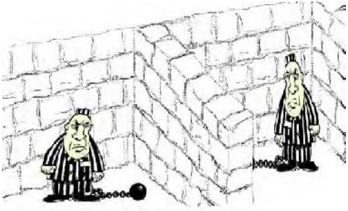
Self-interested Agents

- What does it mean to say that an agent is self-interested?
  - Not that they want to harm others or only care about themselves
  - Only that the agent has its own description of states of the world that it likes, and acts based on this description
- Each such agent has a utility function
  - Quantifies degree of preference across alternatives
  - Explains the impact of uncertainty
  - Decision-theoretic rationality: act to maximize expected utility

Prisoner's Dilemma

Each player have two option of Deny or Confess.

$$u_1(D, C) < u_1(C, C) < u_1(D, D) < u_1(C, D)$$



	D	C
D	(-1,-1)	(-10,0)
C	(0,-10)	(-8,-8)

A Large Collective Action Game

- Players: $N = \{1, \dots, 10000000\}$
- Action set of each player: $A_i = \{Revolt, Not\}$
- Utility function of each player
  - $u_i(a) = 1$ if $\#\{j | a_j = Revolt\} \geq 2000000$
  - $u_i(a) = -1$ if $\#\{j | a_j = Revolt\} < 2000000$ and $a_i = Revolt$
  - $u_i(a) = 0$ if $\#\{j | a_j = Revolt\} < 2000000$ and $a_i = Not$

Constant Sum Games

Constant sum games also known as games of pure competition.

- Players have exactly opposed interests
- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles $a \in A : u_1(a) + u_2(a) = c$ for some constant c. When $c = 0$ game is called a zero sum game(e.g. pick a hand game).
- Thus, we only needs to store a utility function for one player. In a sense, we only have to think about one player's interests

Matching Pennies

One player want to match while other wants to mismatch

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

Pick a Hand

Hider(player I) may hide a coin in his left hand or two coins in his right hand. Chooser(player II) choose a hand and will get the coins in that hand.

	L	R
L	1	0
R	0	2

Games of Cooperation

- Players have exactly the same interests
- No conflict: all players want the same thing
- $\forall a \in A : u_i(a) = u_j(a)$
- We often write such games with a single payoff per cell
- Why are such games “noncooperative”?

Coordination Game

- Which side of the road you should drive on?

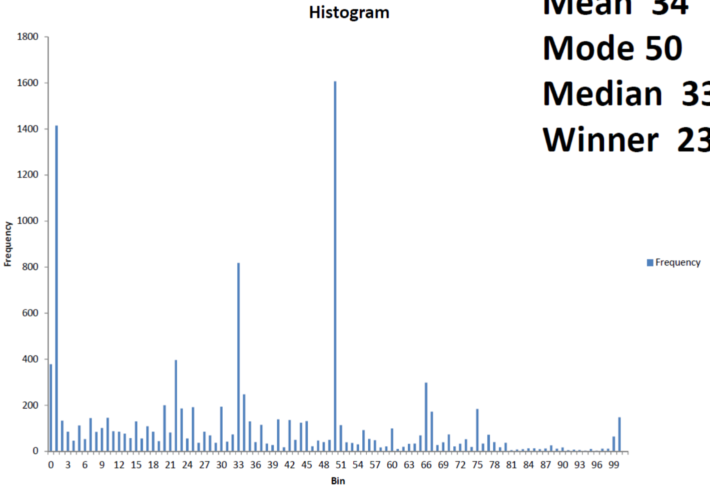
	Left	Right
Left	(1,1)	(0,0)
Right	(0,0)	(1,1)

- Other examples: Battle of sexes, Stag hunt

Keynes Beauty Contest Game

- Each player name an integer between 1 and 100
- The player who names the integer closest to two thirds of the average integer wins a prize, the other get nothing
- Ties are broken uniformly at random

Online course: more than 10000 players:



2012 GTOC

Mean 34

Mode 50

Median 33

Winner 23

Best Response

- If you knew what everyone else was going to do, it would be easy to pick your own action
- let $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$. And $a = (a_{-i}, a_i)$

Definition (Best Response)

$a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i : u_i(a_{-i}, a_i^*) \geq u_i(a_{-i}, a_i)$

Nash Equilibrium

- Really, no agent knows what the others will do
- What can we say about which actions will occur ?
- Idea: look for stable action profiles

Definition (Nash Equilibrium)

$a = (a_1, \ldots, a_n) \in A$ is a (pure)Nash equilibrium iff
$\forall i : a_i \in BR(a_{-i})$

Nash Equilibrium in Examples

	Left	Right
Left	(1,1)	(0,0)
Right	(0,0)	(1,1)

	H	T
H	(1,-1)	(-1,1)
T	(-1,1)	(1,-1)

	B	F
B	(2,1)	(0,0)
F	(0,0)	(1,2)

	D	C
D	(-1,-1)	(-4,0)
C	(0,-4)	(-3,-3)

Strategies

From now we will call actions of each player (i.e. elements of A_i) a pure strategy. We will face mixed strategies later in the course.

Domination

- Let s_i and s'_i be two strategies for player i and let S_{-i} be the set of all possible pure strategy profiles for the other players

Definition

s_i strictly dominates s'_i if $\forall s_{-i} \in S_{-i} : u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

Definition

s_i weakly dominates s'_i if $\forall s_{-i} \in S_{-i} : u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Equilibria and Dominance

- If one strategy dominates all others, we say it is dominant.
- A strategy profile consisting of dominant strategies for every player must be a Nash equilibrium
- An equilibrium in strictly dominant strategies must be unique.

	D	C
D	(-1,-1)	(-4,0)
C	(0,-4)	(-3,-3)

Pareto Optimality

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
- In this cane, it seems reasonable to say that o is better than o'
- We say that o Pareto-dominates o'

Definition (Pareto Optimality)

An outcome o^* is Pareto-optimal if there is no other outcome that pareto-dominates it.

- Can a game have more than one Pareto-optimal outcome?
- Does every game have at least one Pareto-optimal outcome?